**Basic Principles of Deductive Logic - PART 2**

**Part Two:**

Remember that in the last section we said that there are four ways that any two of the four statements - A, E, I, and O - can be related in opposition. In other words, any one of these statements can be said to be opposite to another of them in any one of four different ways. They can be *contradictory* to one another, they can be *contrary* to one another, they can be *subcontrary,* and *subalternate*.

Previously we studied the first two of the four kinds of opposition between categorical statements. We studied contradiction and contrariness. Now, we will discuss subcontrariness and subalternation, the last two of the four kinds of opposition.

**A. __The Rule of Subcontraries.**

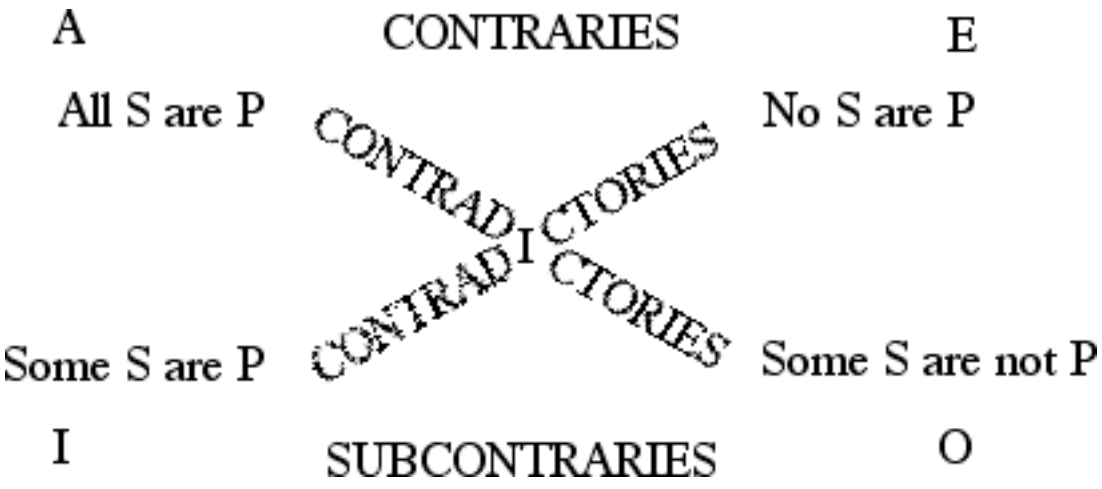
***The Rule of Subcontraries: Two statements are subcontrary if they are both particular statements that differ in quality.***

The difference between contraries and subcontraries is that, while two contrary statements are both universal, two statements that are subcontrary are both particular. Two statements that are subcontrary are both particular, but one is affirmative and one is negative.

Of the four statements we've discussed, A, E, I, and O, which one is particular and affirmative? "Some S are P," the I statement, is particular and affirmative. Of the four statements, which one is particular and negative? "Some S is not P," the O statement, is particular and negative. Therefore, the I and the O statements are subcontrary.

Let's take a look at the *Square of Opposition* (Fig. 2.-1). You can see that the two bottom statements, I and O, are particular but differ in quality.

Figure 2 - 1



Like contraries, but unlike contradictories, there is only one combination of statements that are subcontrary. The two contrary statements are A and E. The two subcontrary statements are I and O.

__**Third Law of Opposition:**

***Subcontraries may at the same time both be true, but cannot at the same time both be false.***

If one is false, the other must be true. If one is true, then the other may be either true or false. Look at the following statements:

I: Some S are P.
O: Some S is not P.

Example: “Some men are mortal” and “Some men are not mortal.”

__**The Rule of Subalterns:**

***Two statements are subalternate if they have the same quality, but differ in quantity.***

They are propositions with the same subject, predicate and copula, one of which is universal and the other particular.

Unlike contradictories, contraries and subcontraries, subalterns are really not opposite to one another. But they do have a particular logical relationship with one another that helps to complete the Square of Opposition.

Whereas there was only one combination of statement that we found to be contrary and subcontrary, there are (like contradictories) two combinations of statements that are subalternate.

We see in Fig. 2 - 2 that there are two pairs of statements that have the same quality but differ in quantity. First, the A statement and the I statement have the same quality (they are both affirmative), but they differ in quantity (A is universal, while I is particular). A and I, therefore, are subalterns.

Second, we notice that the E statement and the O statement have the same quality (they are both negative), but they differ in quantity (E is universal, while O is particular). E and O, therefore, are subalterns.

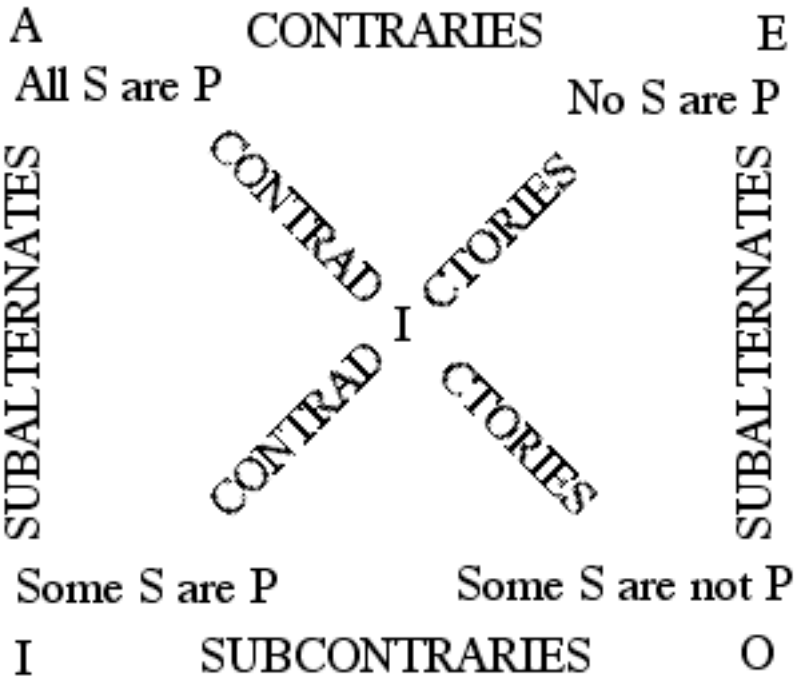
__**The Fourth Law of Opposition:**

***Subalterns may both be true or both be false. If the particular is false, the universal is false; if the universal is true, then the particular is true; otherwise, their status cannot be determined.***

In other words, when it comes to A and I statements, if “Some S are P” is false, then we know that “All S are P, is false. And if “All S are P” is true, then we know that “Some S are P” is true. It also works with E and O statements, since they, too, are subalterns. If “Some S are not P” is false, then “No S are P” must be false. And if “No S are P” is true, then “Some S are not P” must be true.

When we say, in the Fourth Law of Opposition, “otherwise, their status is indeterminate,” we mean (in the case of A and I statements) if “All S are P” is false, we cannot know whether “Some S are P” is true or false. And if “Some S are P” is true, we cannot know whether “All S are P” is true or false. And (in the case of E and O statements, which are also subalterns), if “No S are P” is false, we cannot know whether “Some S is not P” is true or false, and if “Some S is not P” is true, we cannot know whether “No S are P” is true or false.

Fig. 2 - 2:




**Exercises:**

29. Tell whether the following pairs of statements are subcontrary to one another by writing an ‘S’ next to them.

All logic problems are difficult
Some logic problems are difficult.

No logic problems are difficult.
All logic problems are difficult.

Some logic problems are difficult.
No logic problems are difficult.

Some logic problems are not difficult.
All logic problems are difficult.

All logic problems are difficult.
No logic problems are difficult.

No logic problems are difficult.
Some logic problems are not difficult.

30. Can both I and O statements be true at the same time?

31. Can both I and O statements be false at the same time?

32. Tell which of the pairs in #29 are subalternate statements.

33. T F Two statements are subcontrary if they differ from each other in quality and are both particular.

34. T F The A statement and the I statement differ in quality and quantity.

35. T F The statements "All S are P" and "Some S are P" can both be true at the same time.

**B. Distribution of Terms**

**__What is Distribution?**

***Distribution is the status of a term in regard to extension.***

All the statements (A, E, I and O) have a subject. The subject of a statement is the term the statement is about. In the statement, "All S are P," S is the subject. In the statement, "All men are mortal," *men* is the subject.

In addition, all of the statements have a predicate. A predicate is the term we use to say something about the subject. In the statement "All S are P," P is the predicate. In the statement "All men are mortal," *mortal* is the predicate.

We want to know whether the terms used as subject and predicate in each one of the four statements are ***distributed***. When we say that a term is distributed, we mean that the term refers to all the members of the class of things denoted by the term. When we

use the term *man* in a statement, for example, are we referring to it universally - in other words, are we referring to all men? Or are we referring to it particularly - are we referring to only some men? If we are using it universally, we say it is distributed.

When we say *mortal* in a statement, are we using it universally - are we referring to all mortal things? Or are we using it particularly - are we referring only to some mortal things?

We say that a term is distributed when it is used universally - if it refers to all the members of the class denoted by the term. If it is used particularly - if it does refer to only some members of the class denoted by the term, then we say it is *undistributed.*

**__Distribution of the Subject - Term**

***The subject-term is distributed in statements whose quantity is universal and undistributed in statements whose quantity is particular.***

Determining the distribution of the subject-term is easy because the quantifier (*All, Some, No and Some ... not)* tells us all we need to know. If it says "All S are P," we know it refers to **all** S's. It refers to all the members of the class it denotes. If we say "All men are mortal,m" we know it means **all** men. It refers to all the members of the class it denotes. A subject-term in an A statement, then, is taken universally, and is therefore distributed. The same goes for the E statement. It says "No S are P." to how many members of the class denoted by S does this E statement refer? To all of them. To say "No S is P" is the same as saying "All S is not P." In other words the subject-term of the E statement is taken universally and is therefore distributed.

Likewise, when we say "Some S is P," we are obviously not referring to all S's, only some of them. And when we say "Some men are mortal," we are only referring to some men, not all of them. In both of these cases, the subject-terms are undistributed.

The O statement too, "Some S is not P," obviously has a subject term that is not universal and therefore is undistributed.

In the case of the subject-term, the, the quantifier tells us all we need to know. We can see how distribution works with the subject-term in the following diagram:

**DIAGRAM OF THE DISTRIBUTION OF TERMS IN A, E, I, AND O STATEMENTS**

| Type of sentence | Subject-Term |
|---|---|
| **A** | **Distributed** |
| **E** | **Distributed** |
| **I** | **Undistributed** |
| **O** | **Undistributed** |

## __Distribution of the Predicate-Term

***In affirmative propositions the predicate-term is always taken particularly (and therefore undistributed) and in negative propositions the predicate is always taken universally (and therefore distributed).***

**Distribution of the Predicate-Term in A statements.**

Example: All men are animals.

We know we are talking about all men. Are we talking about all animals? Obviously not. The predicate-term is therefore taken particularly, and is therefore undistributed.

**Distribution of the Predicate-Term in E statements.**

Example: No man is a reptile.

As in A statements, the subject of an E statement is universal and therefore distributed. But what about the predicate? Notice that it is logical to infer from "No man is a reptile," that "All reptiles are not men." We are taking reptiles universally, and therefore it is distributed.

**Distribution of the Predicate-Term in I statements.**

Example: "Some dogs are vicious things."

First, we are only talking about some dogs, so the subject is undistributed. Also, we are talking only about some vicious things. There are other vicious things (wolverines, tasmanian devils, etc.) that are not dogs. The predicate *vicious things* is particular and therefore undistributed.

**Distribution of the Predicate-Term in O statements.**

Example: "Some men are not blind."

We see that the subject-term is not distributed. We know we can't say that all men are not blind (only some of them are not blind). But these *some men* who are *not blind* - are they excluded from only part of the class of blind things or are they excluded from the entire class? The some men who are not blind are, of course, excluded from the whole class of blind things. Therefore in the O statement, we are taking P universally. It is therefore distributed.

**DIAGRAM OF THE DISTRIBUTION OF TERMS IN A, E, I, AND O STATEMENTS**

| Type of sentence | Subject-Term | Predicate-Term |
|---|---|---|
| **A** | **Distributed** | **Undistributed** |

| | | |
|---|---|---|
| **E** | **Distributed** | **Distributed** |
| **I** | **Undistributed** | **Undistributed** |
| **O** | **Undistributed** | **Distributed** |

**Exercises:**

36. T F (There is no #36.)

37. T F The subject-term is distributed in statements whose quantity is universal.

38. T F The subject-term is undistributed in statements whose quantity is universal.

39. T F The subject-term in the I statement is undistributed.

40. T F In negative propositions, the predicate is always taken universally.

## C. Obversion, Conversion, and Contraposition

In logic, the way we say two statements are logically the same (even though they may use slightly different words) is by calling them *logically equivalent.* Equivalent propositions can be converted into each other in various ways.

There are three ways to convert propositions into their logical equivalents: Obversion, conversion, and contraposition.

__**Obversion**. To obvert a sentence, you must do two things:

1. Change the quality of the sentence.
2. Negate the predicate.

**To change the quality** is easy. If the statement is affirmative, you simply make it negative. If the statement is negative, you simply make it affirmative. But be careful. Do not change the quantity of the statement. For example, if you say, “All S are P,” you change it to “No S are P.”

Here are a few examples:

All S are P --------> No S are P
No S is P --------> All S is P
Some S are P ------> Some S is not P
Some S is not P----> Some S is P

**To negate a predicate** is also easy: you simply place a *not* in front of it. If you say, for example, “All S are P,” and, in accordance with step 1, change the quality, you get “No S are P.” Negating the predicate, as step 2 requires, will yield “No S is not P.”

Obversion, unlike conversion and contraposition, works on all four kinds of propositions, A, E, I, and O. in other words, if we obvert any of these four statements, we will get a statement that is logically equivalent to the original.

Once we have applied both step 1 and step 2, we end up with statements that do not look as if they mean the same thing, but they are in fact logically equivalent.

Let's look at the statements we started out with and see what they look like after both steps 1 and 2 have been applied:

| | | |
|---|---|---|
| All S are P | -----> | No S are not P |
| No S are P | -----> | All S are not P |
| Some S are P | -----> | Some S are not non-P |
| Some S are not P | -----> | Some S are not P |

If, for example, we want to overt "all men are mortal," we say "No men are not mortal." Logically, they mean the same thing. And if we want to obvert "No men are gods," we say "All men are not gods." Again, they mean the same thing for the purposes of logic.

__**Double Negation of the Predicate in I statements**

Let's take a close look at the I statement for a moment. Notice that with the I statement, you get two negations in the predicate after you obvert: "Some S are P" gets turned into "Some S is not non-P." this is because, under step 1 of obversion, you have changed the quality from affirmative to negative (which in a particular statement you perform by negating the predicate), and then under step 2, you negate the predicate. In other words, you end up negating the predicate twice.

You can handle this in any one of four different ways: First, you can simply have two *nots* in the statement, right next to each other. Secondly, you can make the *not* directly in front of the predicate (i.e. the second not) a *non*, which means the same thing, but can sometimes sound better. Thirdly, you can incorporate the second negation in the predicate word itself by placing an *im*, and *um*, an *in*, or an *ir* at the beginning of the word you are using in the predicate. For example, if the original predicate was *mortal*, you could take care of the second negation by using the word *immortal*.

Be careful that you do not negate the predicate term by using an antonym.

__**Double Negation.** How do you apply step 2, which involves putting a *not* in front of the predicate, if there is already a *not* there? You can apply one of the first three ways of negating the predicate of an I statement, but sometimes this can sound rather awkward. For example, obverting "Some men have brown hair" to "Some men do not have non-brown hair" simply doesn't sound right. The solution to this difficulty lies in applying the logical rule of **double negation.**

***The rule of double negation says that a term which is not negated is equivalent to a term that is negated twice (and vice-versa).***

In other words, “not not P” is logically equivalent to “P.”

In O statements, if we do not apply double negation, we would end up with a triple negation, “Some S is not not not P.” We can get rid of two *nots* by applying double negation, yielding, “Some S is not P,” which of course, is the same statement with which you began.

__**Conversion.** Conversion is even easier than obversion, since it involves only one step. It is as follows:

Interchange the subject and the predicate.

Here are the ways in which sentences are converted:

| | | |
|---|---|---|
| No S are P | -----> | No P are S |
| Some S are P | -----> | Some P is S |

Notice that we have converted only the E statement and the I statement. That is because conversion only yields a logically equivalent statement with these two kinds of statements. For example, “No men are gods” is the same as “No gods are men.” And, “Some animals are friendly things” is the same as “Some friendly things are animals.”

__**Contraposition.** Contraposition, the third method of converting statements into their equivalents, is accomplished in three steps:

1. Obvert the statement.
2. Convert the statement.
3. Obvert the statement again.

Only the A and O statements can be converted in this way.

Example: Original sentence: “All men are mortal.”

Step 1, obvert: No men are non-mortal
Step 2, convert: No non-mortals are men
Step 3, obert:All non-mortals are non-men.

| | | | |
|---|---|---|---|
| A | All S are P | -----> | All non-P is non-S |
| O | Some S is not P | -----> | Some non-P is S. |

**Exercises:** (We’ll practice this more in class.)

41. T F The three ways statements can be converted into their logical equivalents are by obversion, conversion, and subalternation

42. T F Obversion can be performed on all four kinds of statements.

43. T F The statement “All lobsters are angry” and the statement “No lobsters are angry” are contrapositive.

44. T F There are only two steps involved in obversion.

**D. What is Deductive Inference?**

Up to this point, we’ve discussed *proposition*, which is the verbal expression of judgment. We discussed how they are logically opposed and how they are logically equivalent. We also discussed how terms are distributed in each kind of proposition. Earlier we discussed simple apprehension (or *term).* Simple apprehension is the first logical operation of the mind, judgment is the second. Now, we will turn to the study of the third logical operation of the mind: syllogism, which is the verbal expression of deductive inference.

| Mental Act | Verbal Expression |
|---|---|
| Simple Apprehension | Term |
| Judgment | Proposition |
| **Deductive Inference** | **Syllogism** |

__**Reasoning.** Deductive inference is one kind of reasoning. Reasoning is defined as follows:

***Reasoning is the act by which the mind acquires new knowledge by means of what it already knows.***

When we reason, we take truths that are already known to us and, by the use of reasoning, arrive at another truth. There are two kinds of reasoning:

1. Deduction (i.e. Deductive Inference)
2. Induction.

Remember that we are studying only deduction in these lessons.

Let’s look at the following argument:

All men are mortal.
Socrates is a man.
Therefore, Socrates is mortal.

There are three acts which occur in our minds when we make an argument like this. First, we perceive the first premise (“All men are mortal”) as being true. Secondly, we

perceive the second premise ("Socrates is a man") is also true. These two steps are together called the *antecedent.* The word *antecedent* is made up of two Latin words: *ante,* which means *before,* and *cedre,* which means *to go*. These first two steps -- the recognition that each of the two premises is true -- *go before* or precede in the act of reasoning.

Each one of the first two steps is an act of judgment, which is, as we said, the second operation of the mind. The third step is an act of deductive inference, the third logical operation of the mind. This third step takes place when we realize that, given the truth of the two premises ("All men are mortal" and "Socrates is a man"), the conclusion ("Socrates is a mortal") must also be true. Our minds stoop, or *conclude* at this third step, which is why this final statement is called the *conclusion.* The conclusion is the *consequent* in our reasoning.

***Deductive inference is the act by which the mind establishes a connection between the antecedent and the consequent.***

As we said, deductive inference is the mental act, and it corresponds to the verbal expression we call a *syllogism*.

***A syllogism is a group of propositions in orderly sequence, one of which (the consequent) is said to be necessarily inferred from the others (the antecedent).***

A syllogism will always contain who premises and a conclusion The premises are the antecedents and the conclusion is the consequent.

**Validity.** All reasoning presupposes what we may call the ***Essential Law of Argumentation:***

***If the antecedent is true, the consequent must also be true.***

All valid syllogisms are governed by this law. A valid syllogism is one that contains a conclusion that logically follows from the premises. We see this law in operation by once again looking at the argument:

ALl men are mortal
Socrates is a man.
Therefore, Socrates is a mortal.

We can see that if the antecedent is true, then the statement "Socrates is mortal" must also be true.

This rule has two corollaries to it:

1. If the syllogism is valid and the consequent is false, then the antecedent (i.e. one or both of the premises) must be false.

2. In a valid syllogism with a true consequent, the antecedent is not necessarily true (i.e. one or both of the premises may still be false).

Example of corollary 1:

All men are sinners.
My dog Spot is a man.
Therefore, my dog Spot is a sinner.

This syllogism is valid. By saying it is valid, however, we simply mean that **if** the premises are true, the conclusion must also be true. But the conclusion is false. By applying corollary 1, we see that, if the conclusion is false, one or both of the premises must be false. in this particular argument, we see the problem right away: the second premise is obviously false.

Example of corollary 2:

All vegetables are philosophers.
Socrates is a vegetable.
Therefore, Socrates is a philosopher.

In this syllogism, we notice that the conclusion is true: Socrates was, indeed, a philosopher. But we know from corollary 2. that just because the conclusion is true doesn't mean that the antecedent must be true. Indeed, we see in this argument that even though the consequent (conclusion) is true, neither of the premises are.

__**Terms in a Syllogism.** There are three terms in a syllogism: The *major term,* the *minor term*, and the *middle term.* They are arrayed in a valid syllogism as follows:

Major Term: The major term is the predicate of the conclusion.
Minor Term: The minor term is the subject of the conclusion.
Middle term: The middle term is the term that appears in both premises, but not in the conclusion.

All men are mortal.
Socrates is a man.
Socrates is mortal.

We see in this syllogism that there are six terms used, but some of them are the same. There are actually three terms used twice each. Using the definitions of the three kinds of terms above, we can determine what the minor, major, and middle terms are in this argument.

Major Term: mortal
Minor Term: Socrates
Middle Term: men

In addition to the labels we attach to the terms themselves, there are also labels we attach to the premises in an argument. One of the premises we call the *major premise;* the other we call the *minor premise*.

The major premise is the premise that contains the major term.
The minor premise is the premise that contains the minor term.

__**Proper Formation of a Syllogism** in the example syllogism, we found that the major premise is the first one. It is very important to remember that the major premise should always be put first in a syllogism. We say a syllogism is properly formed if the major premise is first, the minor premise second, and the conclusion third.

__**Exercises**

45. Fill out the following chart showing the three aspects of logic (Review)

| **Mental Act** | **Verbal Expression** |
|---|---|
| | |
| | |
| | |

46. What is the definition of deductive inference?

47. What is the definition of syllogism?

48. Identify the antecedents and the consequents in the following syllogisms (Keep in mind that every premise is considered an antecedent and that the consequent is the same as the conclusion.):

All men are mortal.
Socrates is a man.
Therefore, Socrates is mortal.

All birds are able to fly.
The ostrich is a bird.
Therefore, the ostrich is able to fly.

All fish can live out of water.
A dog is a fish.
Therefore, a dog can live out of water.

No ducks are birds.
A Mallard is a duck.
Therefore, a mallard is not a bird.

49. Explain how to distinguish each of the following: Major term, Minor term, Middle term.

50. In a syllogism, which premise is the major premise?