Page 8: Formal Logic

Introduction to Formal Logic

All arguments fall into two fundamental categories; deductive and inductive. Using a deductive pattern will greatly increase the logical power of your argument, if you already have reasons which are true.

Deductive Arguments

Defined: An argument where IF the premises were true, then the conclusion would have to be true.

An interesting controversy regarding deduction:

Note: The following definition for deduction is preferred by many who teach logic and critical thinking:

"Any argument where the intent is for the premises to lead to the conclusion with certainty."

Intention is very important in this definition, because it includes certain argument forms which the first definition excludes (see Formal Fallacies). These additional forms are fallacious (i.e., the premises do not actually lead to the conclusion with certainty, but the author erroneously believes that they do). We call these fallacious forms "deductively invalid."

This author prefers the first definition because it does not require one to ascertain intention, but rather focuses on the actual logical structure of the argument. Either it is actually valid (hence deductive) or it is invalid (hence inductive). This is consistent with the definition given in A Dictionary of Philosophy by Antony Flew

A valid argument in which it is impossible to assert the premises and to deny the conclusion without thereby contradicting oneself.

Notice that the first definition makes the words "deductive" and "valid" synonyms, where the second definition requires the addition of "valid" or "invalid" to indicate the quality of the argument.

NOT just any argument where the premises ARE true, because inductive arguments can have premises that are true.

This definition sounds a little strange because one can have a deductive argument where the premises are not true and it is still deductive. If those premises WERE true, then the conclusion would have to be true.

Example: If Bill Clinton is president, then dogs can fly. Bill Clinton is president. Therefore, dogs can fly.

The first premise is true, but the second one isn't. But, IF both premises WERE true, then there is no possible way that the conclusion could be false. It is the FORM that guarantees this.

If the premises are true, and the form is deductive, then the probability of the conclusion being true is 100%.

Deductive arguments draw out conclusions which are already contained in the premises.

Here is an analogy to help you to understand this point. Refrigerators preserve food. If the food that goes into the refrigerator is good, then the refrigerator maintains that goodness. If the food is already rotten when it goes into the refrigerator, then the food remains rotten. Likewise, the truth of reasons is preserved by a deductive argument form. If your reasons are true, then a true conclusion is guaranteed.

Valid arguments Defined: Any argument where IF the premises were true, then the conclusion must be true.

We say an argument is "valid" if and only if it is a deductive form. The truth (or falsity) of the premises is irrelevant.

It is possible for an argument to be valid and untrue at the same time. Let's look at the following argument to illustrate: "If it is raining, then it is cloudy. It is raining. Therefore, it is cloudy." Look outside. It may not be raining right now. But the argument is still a valid deductive argument, because IF the premises WERE true, then the conclusion would have to be true. In other words, there is no possible way for the premises to be true and the conclusion to be false. If it's not raining right now, pretend. Can you imagine it raining without at least a small cloud? No, that's not possible. So we know that the argument is valid.

Invalid arguments are all those which are not valid. That is to say, all inductive arguments are (deductively) invalid.

Sound arguments

Defined: Any valid argument which has premises that are true.

When analyzing an argument, there are two features which you must look at. First, determine whether or not the argument is valid. In other words, is the argument a deductive form? Second, determine whether or not the argument has premises which are true. If both criteria are met, then the argument is sound. This means there is no possible way that the conclusion can be false.

(Note: The word "cogent" means the same thing as "sound.")

Inductive Arguments

Defined: Any argument which is intended to give some support, but not definitive support for the conclusion.

An important characteristic of inductive arguments is that it is always possible to have an argument where the premises are true, but the conclusion is false. One hopes that the conclusion is true, but it might not be. Probability is always a factor with inductive arguments. The certainty of the conclusion being true is always less than 100%.

Example: All the presidents of the United States have been men. Therefore, it is likely that the next president will be a man.

The premise in this argument is true. However, it is easy to see that it is possible that a woman will be the next president. So, even though the premise is true, it is possible for the

conclusion to be false.

Example: 75% of potential voters polled say they are voting for Donaldson. Therefore, it is likely that Donaldson will win.

The premise deals with statistics, in this case a sample of voters. It is always possible that something went wrong with the poll, so it is not certain that Donaldson will win.

When the premises are used as a base from which the conclusion makes a projection, then the argument is inductive.

Remember, all inductive arguments are (deductively) invalid. (see validity)

Five types of inductive arguments

Generalizations: extending observations of some to an entire class. Analogy: showing that something easily understood is similar to something more complex in relevant ways.

Statistical: inferring that the qualities of the data adequately represent the qualities of the population.

Higher induction: intellectual reasoning that shows certain things are likely to be true.

Hypothesis: systematic process by which a theory is inductively confirmed or falsified.

Conditional Claims

Conditional claims are also called hypothetical claims.

The basic form of the conditional claim is, "IF_____, THEN_____." For instance, "If the sun is shining, then it will be warm." Every conditional claim contains two independent clauses. One independent clause follows the "if." The other independent clause follows the "then."

The conditional claim can be put into symbolic form; . This symbolic form is read, "If P, then Q." The P and the Q represent independent clauses.

It is called hypothetical because we don't know from looking at the claim itself whether or not the sun is currently shining, or whether or not it is warm. We only know one thing: that whenever that the sun is shining, that it will be warm.

A COMMON ERROR is to think of the conditional claim as an entire argument. An argument requires a minimum of two sentences. The conditional claim is only one. So it can't be an entire argument.

Antecedent and consequent The part of the sentence that comes after the "if" and before the "then" is called the antecedent. For instance, "the sun is shining"

The part of the sentence that comes after the "then" is called the consequent. For instance, "it will be warm."

Necessary and Sufficient conditions

Let's use an example to illustrate these concepts. "If it is raining, then it is cloudy." Let's discuss all aspects of this claim to make sure there is no confusion. First, the claim is true. Rain comes from clouds. Now sometimes it is sunny and raining. That's an unusual experience, but it happens. However, there must be a cloud somewhere, because that's the only place rain comes from. So whenever the first condition is present (it is raining), the second condition is certain to be present (it is cloudy).

The consequent is also called the "necessary condition." Remember that the consequent follows "then." That means there is always a relationship between the first part of the sentence (If____,) and the second part (then___). And the relationship works this way. Whenever the first condition is present, then the second condition MUST be present. In other words, it is NECESSARY that the second condition be present. For instance, every time that it rains, it is NECESSARY that it be cloudy. It has to be cloudy in order for there to be rain (at least a little bit cloudy).

The antecedent is also called the "sufficient condition." This means that whenever this condition is present, for instance, "it is raining," that this is SUFFICIENT to guarantee that the consequent is present. In other words, that is all it takes to ensure the consequent. The presence of rain is all it takes to guarantee that it is cloudy. Or, rain is sufficient for clouds. So, whenever rain is present, clouds are present - guaranteed.

Does it work both ways? NO. Here's why.

First, think about it for a moment. Does it always rain when there are clouds? Of course not. Sometimes it's cloudy and there is no precipitation. Other times it might snow, sleet, or hail. The presence of clouds in no way guarantees that it is raining. In other words, saying "If it rains, then it is cloudy," is not at all the same as saying "If it is cloudy, then it is raining." Conditional claims only work in one direction.

Every conditional claim has one sufficient condition and one necessary condition. The antecedent = the sufficient condition. The consequent = the necessary condition.

Beyond the grammatical structure of the conditional claim, it is important to evaluate the actual relationship between the clauses. Is the stated relationship actually the case?

For instance, "If Bill Clinton is president, then the moon is full." Bill Clinton is president and sometimes the moon is full. But is Bill Clinton's being president enough to make the moon full? Of course not. So even though we can randomly plug independent clauses into conditional claims, it does not mean the conditions are actually sufficient or necessary. Again, one must evaluate the logical structure as well as the truth of the content.

Valid Patterns Using Conditional Claims

Introduction Conditional claims can act as the anchor of very powerful arguments. The following basic forms can become the basis of fairly elaborate arguments. Though the content of the claims can be very different, there are only a few basic patterns. We will use P, Q, and R to represent various independent clauses.

Conditional claims don't tell us anything about the actual presence or absence of the conditions. That's why we call them hypothetical. The following argument patterns make additional assertions that allow us to draw conclusions with certainty. It is the specific

relationship between the assertions and clauses within the conditional claims that allow us to make such strong conclusions.

Modus Ponens

Modus ponens literally means "the mode of affirming."

If P, then Q. P. Therefore Q.

Example. If it is raining, then it is cloudy. It is raining. Therefore, it is cloudy.

How does it work? Recall that "sufficient" means "all that is required." So we know that the sufficient condition "it is raining" is all that is required to guarantee that "it is cloudy." And whenever "it is raining" it is necessary that "it is cloudy." Now what we are doing is making the definite assertion that indeed "it IS raining." Knowing that the sufficient condition actually is present allows us to draw the conclusion "It is cloudy" with certainty.

If you're looking outside right now and thinking, "Yeah, but it's not raining!" review validity.

It's called the mode of affirming because it affirms Q by affirming P.

Modus Tollens

Modus tollens literally means "the mode of denying."

If P, then Q. It is not the case that Q. Therefore, it is not the case that P.

Example. If it is raining, then it is cloudy. It is not cloudy. Therefore, it is not raining.

How does it work? Recall that "necessary" means that whenever the antecedent is present, the it is necessary that the consequent be present. So it logically follows that if the necessary condition (the consequent) is NOT present, that the sufficient condition (the antecedent) couldn't be present either. In our example, you'll remember that there has to be at least a small cloud for there rain, so we know that whenever there are no clouds, then it is guaranteed that there is no rain.

It is called the mode of denying because it denies P by denying Q.

Chain Argument (Hypothetical Syllogism)

If P, then Q. If Q, then R. Therefore, If P, then R.

Example. If it is raining, then it is cloudy. If it is cloudy, then I'll be sad. Therefore, If it is raining, then I'll be sad.

How does it work? Notice the "link" between the two premises. The necessary condition in the first premise is also the sufficient condition in the second premise. This makes the "chain." So it follows that if the antecedent is true in the first sentence, then the consequent in the second sentence must be true.

Why is it called the "chain argument"? The premises have an independent clause in common. That independent clause must be the consequent in one premise and the antecedent in the other. This common clause makes the "link" which forms the chain.

Why is it called a "hypothetical syllogism"? Notice that all of the sentences are hypothetical claims. Even the conclusion is a hypothetical claim. A "syllogism" is a form of argument featuring two premises and a conclusion. The name "hypothetical syllogism" is a bit of a misnomer because these arguments can have more than two premises.

Formal Argument Patterns Using Disjunctive Claims

Introduction Disjunctions are compound claims which are made up of "disjuncts" The disjuncts are separated by the word "or" or some synonym.

Inclusive and Exclusive "or"

The word "or" is more complex than many first realize. It has two meanings. This isn't always apparent from the context of the sentence, so one must be careful with arguments featuring disjunctive claims.

Exclusive "or"

Example: "Either I will go to Yale or I will go to Stanford." We can see from the context of this sentence that the person is going to one or the other but not both.

Exclusive "or" means, "Either A or B, but not both A and B."

Inclusive "or"

Example: "Either Bob will be at the party or Donna will be at the party." We can see from the context of this sentence that it is possible that both Bob and Donna could show up to the party.

Inclusive "or" means, "Either A or B, or BOTH A and B."

Valid Disjunctive Syllogisms

The inclusive / exclusive aspect of "or" means that there are two disjunctive syllogisms that are valid and two that are invalid (formal fallacies). In this section we will cover the valid forms.

P or Q. It is not the case that P. Therefore, Q.

For instance, "Either I will go to Yale, or I will go to Stanford. I will not go to Yale. Therefore, I will go to Stanford."

P or Q. It is not the case that Q. Therefore, P.

For instance, "Either I will go to Yale, or I will go to Stanford. I will not go to Stanford. Therefore, I will go to Yale."

Notice that in each of the valid forms the second premise denies one of the disjuncts. Valid disjunctive syllogisms always deny one of the disjuncts. Valid DS are limited to these two because of the exclusive / inclusive problem. To learn more about this go to invalid disjunctive syllogisms.

Valid Disjunctive Syllogism

Formal Fallacies

Introduction An argument is fallacious when the reasons offered fail to warrant acceptance of the conclusion. The fallacies in this section are called formal fallacies because problems with the FORM or structure of the argument mean that the conclusion is not certain. In other words, whenever you encounter an argument using one of these forms, there is cause to doubt the conclusion.

Affirming the Consequent

If P, then Q. Q. Therefore, P.

Example: If it is raining, then it is cloudy. It is cloudy. Therefore, it is raining.

Recall that whenever the antecedent is present, the consequent necessarily follows. But it doesn't work the other way around. The consequent's presence tells us nothing about the antecedent's presence. Experience tells you that there are times when it is cloudy (the consequent), but it is NOT raining. This shows that it is possible for the premises of this type of argument to be true and the conclusion to be false. Hence, it is not a valid argument form.

Denying the Antecedent

If P, then Q. It is not the case that P. Therefore, it is not the case that Q.

Example: If it is raining, then it is cloudy. It is not raining. Therefore, it is not cloudy.

The antecedent's presence guarantees the consequent's presence. But the absence of the antecedent does not tell us anything about the consequent. Remember that rain is sufficient to guarantee clouds. But the absence of rain doesn't mean that there are no clouds. We can see that the premises of this type of argument do not lead with certainty to the conclusion. Therefore, the form is a fallacy.

Reverse Chain

If P, then Q. If Q, then R. Therefore, if R, then P.

The reason this is a fallacy is the same reason that affirming the consequent is a fallacy. It does not follow that if the necessary condition of one premise is present that the sufficient condition of another will be. This is working backwards.

Example: If it is raining, then it is cloudy. If it is cloudy, then I'll be sad. Therefore, if I am sad, then it is raining. This incorrectly assumes that the only possible reason for sadness is rain. We know there are other causes of sadness. Rain is just one of them (for this person).

Invalid Disjunctive Syllogisms

P or Q. P. Therefore, it is not the case that Q.

Example: Either Bill will be at the party or Donna will be at the party. Bill is at the party. Therefore, Donna is not at the party.

P or Q. Q. Therefore, it is not the case that P.

Example: Either Bill will be at the party or Donna will be at the party. Donna is at the party. Therefore, Bill is not at the party.

At first glance these seem alright. However, it is possible, given the meaning of the word "or" that Donna and Bill could BOTH be at the party. So it is possible for the premises to be true and the conclusion to be false.